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NONLINEAR EFFECTS IN MEDIA HEATED BY ELECTROMAGNETIC RADIATION

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The features of thermal processes in dielectric absorbing media heated by high-frequency electromagnetic radiation are investigated. It is shown that the temperature dependence of the absorption coefficient of the electromagnetic radiation can result in the development of nonlinear effects: progressive heating in a regime with peaking and localization of the heating region, heating in a regime of lightening of the medium, and formation of quasistationary temperature waves.

1. The heating of media by electromagnetic radiation is determined by the density of the heat sources Q, which, in the one-dimensional case, has the form

$$Q = 2\alpha\, q_0 \exp(-2\alpha x)\,, \quad \alpha = \frac{1}{2}\frac{\omega}{c_0}\sqrt{\varepsilon}\,\mathrm{tg}\,\delta\,. \tag{1}$$

The theory of dielectric losses and experimental investigations [1, 2] show that ε and $\tan\delta$ depend on the temperature T. As a consequence, the absorption coefficient and the depth of penetration of radiation into the medium $h = 1/2\alpha$ are also a function of the temperature. For different media, the dependence $\alpha(T)$ can have the form of a monotonically increasing or decreasing function; nonmonotonic dependences with a single or several extreme values are also possible. Certain features of the process of heating of dielectrics by a microwave electromagnetic field in the case where the absorption coefficient increases with the temperature have been investigated in [3, 4].

In the case of a nonmonotonic dependence with an extremum at $T = T_m$, the following approximation is possible:

$$\frac{d\alpha}{dT} = \pm\left|\frac{d\alpha}{dT}\right| \mathrm{sign}\,(T_m - T)\,, \quad \mathrm{sign}\,x = \frac{x}{|x|}\,. \tag{2}$$

From (1) it follows that

$$\frac{dQ}{dT} = 2q_0\frac{d\alpha}{dT}\exp(-2\alpha x)\,(1 - 2\alpha x)\,. \tag{3}$$

It is seen from (3) that when $d\alpha/dT > 0$, when the absorption coefficient increases with the heating, the medium heats up in a regime with peaking (in the region $x < h$, $dQ/dT > 0$) and localization ($dh/dT < 0$), and hence self-accelerating heating of a region confined to the surface $x = 0$ occurs. In this case, in the region $x > h$, $dQ/dT < 0$.

When $d\alpha/dT < 0$, the depth of penetration of radiation into the medium increases with the heating; in this case, $dQ/dT < 0$ at $x < h$ and $dQ/dT > 0$ at $x > h$. In the process of heating, the medium becomes more transparent to radiation, in other words, heating in a regime of lightening of the medium occurs. The heating of the medium occurring in this case would be expected to be more uniform and deep than with the heating realized in the previous case.

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In the case of a nonmonotonic dependence $\alpha(T)$ determined by (2) with the positive sign in the right-hand side (a convex dependence), heating in a regime with peaking and localization occurs in the initial stage where $T < T_m$. At $T = T_m$, the regime of heating changes and it occurs with lightening of the medium. In the case of the negative sign in the right-hand side of (3) (a concave dependence), the situation is opposite.

When $2\alpha x = 1$, i.e., $x = h$, $dQ/dT = 0$; the surface $h = h(T) = h(x, t)$ is a moving one, and we will call it the wave of lightening of the medium. Obviously, on the lightening wave $x = x_s(t)$, $h = h(x_s(t), t) =$ const, and $\alpha = \alpha(x_s(t), t) =$ const. Then

$$\frac{d\alpha}{dt} = \frac{\partial \alpha}{\partial t} + \frac{\partial \alpha}{\partial x_s}\frac{\partial x_s}{\partial t} = 0 .$$

From this expression we can determine the velocity of the illumination wave:

$$V_s = \frac{\partial x_s}{\partial t} = -\frac{\partial \alpha}{\partial t} \Big/ \frac{\partial \alpha}{\partial x_s} . \tag{4}$$

2. The dielectric properties of the medium change as it heats up, which results in a dependence of the dielectric permittivity of the medium on the coordinate. Because of this, it becomes necessary to solve the wave equation for the strength of an electromagnetic field in an inhomogeneous medium. Such a solution can be obtained by the Wentzel–Kramers–Brillouin method [5, 6]. Here the density of the heat sources in weakly absorbing dielectrics ($\tan^2 \delta << 1$) is represented in the form

$$Q = 2q_0\, \alpha(x, t) \exp\left(-\int_0^x 2\alpha(x', t)\, dx'\right).$$

Note that this expression can also be obtained directly from the Umov–Pointing equation determining the density of the heat sources in terms of the electromagnetic-radiation intensity and from the generalized Bouguer–Lambert–Beer law that describes the dissipation of radiation energy in an inhomogeneous absorbing medium:

$$Q = -\frac{\partial q}{\partial x}, \quad dq = -\alpha(x, t)\, q(x, t)\, dx .$$

Combining these relations, we obtain an expression for Q that coincides with the above expression with an accuracy to a factor equal to two, which can be eliminated by overdetermining α.

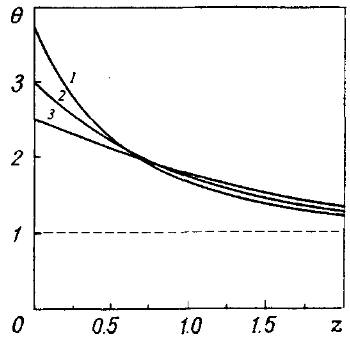
3. The subject of further investigation is the following problem for the heat-conduction equation:

$$\frac{\partial \theta}{\partial \tau} = a_0 \frac{\partial^2 \theta}{\partial z^2} + 2f(z, \tau) \exp\left(-\int_0^z f(z', \tau)\, dz'\right) , \tag{5}$$

$$\frac{\partial \theta(0, \tau)}{\partial z} = 0 , \quad \theta(z, 0) = \theta(\infty, \tau) = 1 . \tag{6}$$

Here $a_0 = 4\lambda T_0 \alpha_0/q_0$, $\theta = T/T_0$, $z = 2\alpha_0 x$, $\tau = t/t_0$, $t_0 = \rho c T_0/\alpha_0 q_0$, $f(z, \tau) = \alpha(x, t)\alpha_0$, and $\alpha(x, t) \equiv \alpha(T)$ is a specified function.

In all probability, problem (5)-(6) cannot be solved analytically. In the case where the medium is heated by a volumetric heat source, the influence of molecular heat conduction is as a rule insignificant; indeed, $a_0 << 1$ for typical values of the parameters. At $a = 0$, the Cauchy problem for an integro-differential



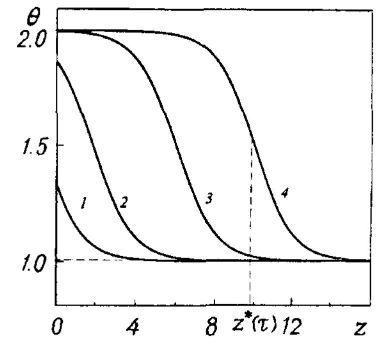


Fig. 1. Temperature distribution at a fixed time for different temperature dependences of the absorption coefficient: 1) $\gamma > 0$; 2) $\gamma = 0$; 3) $\gamma < 0$.

Fig. 2. Dynamics of the change in the temperature profile with time for $\gamma < 0$: 1) $\tau = 0.2$; 2) 1; 3) 3; 4) 5.

equation in which the coordinate z enters as a parameter, follows from (5)-(6). In [6], the solution of an analogous problem for the linear dependence $\alpha = \alpha_0 + \gamma(T - T_0)$ has been obtained.

In accordance with the preceding, we consider three cases:

$$f = f_a = 1\,,\quad f = f_b = 1 + a_1(\theta - 1)\,,\quad f = f_c = 1 - a_1(\theta - 1)\,,$$

$$a_1 = \gamma T_0/\alpha_0\,,\quad \gamma = d\alpha/dT\,, \tag{7}$$

which describe constancy of the absorption coefficient and its increase and decrease with increase in the temperature.

The corresponding solutions of problem (5)-(6) at $a_0 = 0$, obtained by the method described in [6], are as follows:

$$\theta_a = 1 + 2\tau\exp(-z)\,,\quad \theta_b = 1 + \frac{1}{a_1}\frac{1 - \exp(-2a_1\tau)}{\exp z - [1 - \exp(-2a_1\tau)]}\,, \tag{8}$$

$$\theta_c = 1 - \frac{1}{a_1}\frac{1 - \exp(2a_1\tau)}{\exp z - [1 - \exp(-2a_1\tau)]}\,. \tag{9}$$

The special features of solutions (8)-(9) are determined by the parameter a_1, which depends on the initial value of the absorption coefficient and the coefficient of its temperature dependence. When $a_1 \to 0$, the obvious result: $Q_b \to Q_a$, $Q_c \to Q_a$ occurs. Figure 1 shows the dependence $\theta(z)$ for $\tau = 1$ and $a_1 = 0.3$, where curves 1, 2, and 3 correspond to θ_b, θ_a, and θ_c. In the calculations it was assumed that $\rho c = 2\cdot10^6$ J/(m^3·K), $T_0 = 300$ K, $\alpha_0 = 0.1$ 1/m, $q_0 = 10^5$ W/m^2, and $t_0 = 6\cdot10^4$ sec. The course of the curves is evidence in favor of the above conclusions: for $\gamma > 0$, more intense heating of the near zone occurs, and for $\gamma < 0$ the far zone (curve 3) heats up more intensely. As the parameter a_1 increases, this tendency manifests itself more sharply – curve 1 rises sharply at the limit, and curve 3 goes below curve 2 and flattens out gradually.

An analysis of the second expression of (8) shows that with time (for a fixed a_1) progressive heating of the near zone is realized, and for $\tau \to \infty$ localization of the heating region occurs:

$$\theta_b(\tau \to \infty) = 1 + \frac{1}{a_1}\frac{1}{\exp(z) - 1}\,.$$

Thus, for the fixed point z_1 there is a limiting temperature determined only by the parameter a_1. The time of establishment of the asymptotic temperature is of the order of $\tau_\infty \sim \text{const}/a_1$. For example, for $\tau_\infty = 4.605/a_1$ and $a_1 = 1$ the difference between the temperatures θ_b and θ_b ($\tau \to \infty$) is less than 0.01 at $z > 0.01$.

Results of a calculation performed by formula (9) for $a_1 = 1$ and different values of τ are presented in Fig. 2. As the medium heats up, the absorption coefficient decreases and with time a quasistationary temperature wave whose amplitude and velocity tend asymptotically to certain limiting values is formed. The amplitude of the wave is determined from (9) for $\tau \to \infty$: $\theta_s = 1 + 1/a_1$. As follows from (7), this value of θ_s corresponds to the limiting case where $f_c = 0$, i.e., complete lightening of the medium is realized. Thus, the velocity of the temperature wave and the velocity of the lightening wave are the same, and then from (4) with account for the expression $\alpha = \alpha_0 - \gamma(T - T_0)$ (in dimensionless form this is the expression for f_c according to (7)), we obtain

$$V_s = \frac{2a_1}{1 - \exp(-2a_1\tau)} .$$

For $\tau \to \infty$, $V_s = 2a_1$; here the coordinate of the temperature-wave front is $z_s = 2a_1\tau$. The values of θ_s and $V_s = 2a_1$ are attained asymptotically for $\tau \to \infty$; however, it was found that in the case where the condition $2a_1\tau >> 1$ is fulfilled, the amplitude and velocity of the wave have practically limiting values. Thus, for $a_1 = 1$, at the point $z = 0$, $\theta = 0.99\theta_s$ for $\tau = 1.96$, and at the point $z = 1$, $\theta = 0.99\theta_s$ for $\tau = 2.45$. For $\tau = 2$, $V_s \approx 2.037a_1$.

In dimensional form we have

$$T_s = T_0 + \alpha_0/\gamma , \quad V_s = \gamma q_0/(\alpha_0 \rho c) = q_0/\rho c\,(T_s - T_0) .$$

It is not difficult to note that the velocity of the temperature wave satisfies the heat-balance condition; it is proportional to the radiation intensity and inversely proportional to the amplitude of the wave $T_s - T_0$; the amplitude in turn is independent of the intensity and is determined only by the absorption coefficient of the radiation (α_0, γ_0).

4. It was noted above that, in the general case, the dependence $\alpha(T)$ is nonmonotonic and has an extremum (formula (2)). Here, a piecewise-linear approximation of the function $\alpha(T)$ is possible. In dimensionless form this approximation is as follows:

$$f = \begin{cases} 1 & \theta \le 1 , \\ 1 + a_1(\theta - 1) & 1 \le \theta \le \theta_m , \\ a_3 - a_2(\theta - \theta_m) & \theta_m \le \theta \le 1 , \\ a_4 & \theta \ge \theta_1 , \end{cases} \tag{10}$$

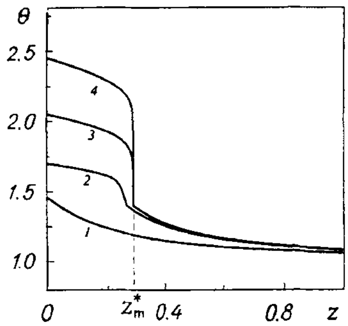
$$a_1 = \frac{\gamma_1 T_0}{\alpha_0} , \quad a_2 = \frac{\gamma_2 T_0}{\alpha_0} , \quad a_3 = \frac{\alpha_m}{\alpha_0} , \quad a_4 = \frac{\alpha_1}{\alpha_0} , \quad \gamma_1 = \frac{\alpha_m - \alpha_0}{T_m - T_0} , \quad \gamma_2 = \frac{\alpha_m - \alpha_1}{T_1 - T_m} .$$

The solution of Eq. (5) at $a_0 = 0$ for the dependence $f(z, \tau)$ determined according to (10) has the form

$$\theta_a = 1 + \frac{1}{a_1} \frac{1 - \exp(-2a_1\tau)}{\exp z - [1 - \exp(-2a_1\tau)]} , \quad 1 \le \theta \le \theta_m ;$$

$$\theta_b = \theta_m + \frac{a_3}{a_2} \frac{\exp[2a_2(\tau - \tau_m)] - 1}{\exp(a_3 z) + \exp[2a_2(\tau - \tau_m)] - 1} , \quad \theta_m < \theta \le \theta_1 ; \tag{11}$$

$$\theta_c = \theta_1 + 2a_4(\tau - \tau_1)\exp(-a_4 z) , \quad \theta > \theta_1 .$$



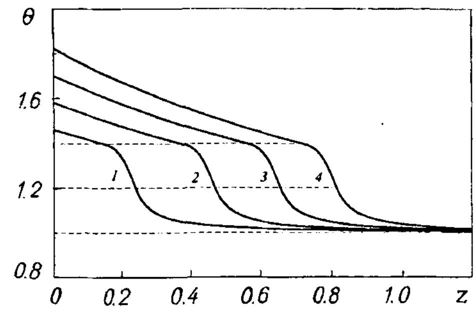


Fig. 3. Temperature distribution for a nonmonotonic temperature dependence of the absorption coefficient: 1) $\tau = 0.1$; 2) 0.25; 3) 0.6; 4) 1; z_m^*, boundary of the localization region of heating.

Fig. 4. Dynamics of a temperature wave in a heat-conducting medium: 1) $\tau = 0.12$; 2) 0.24; 3) 0.36; 4) 0.48.

Here the functions $\tau_m(z)$ and $\tau_l(z)$ are determined, respectively, from the conditions $\theta_a(\tau = \tau_m) = \theta_m$, $\theta_b(\tau = \tau_l) = \theta_l$ and have the form

$$\tau_m = \frac{1}{2a_1} \ln \frac{1 + a_1 (\theta_m - 1)}{1 + a_1 (\theta_m - 1)(1 - \exp z)}, \tag{12}$$

$$\tau_l = \tau_m + \frac{1}{2a_2} \ln \left[1 + \frac{a_2 (\theta_l - \theta_m) \exp (a_3 z)}{a_3 - a_2 (\theta_l - \theta_m)} \right]. \tag{13}$$

Expressions (12) and (13) determine the time of establishment of the temperatures θ_m and θ_l at an arbitrary point of the medium. Solving (12) for the coordinate, we obtain the dependence $z_m(\tau)$, from which it follows that

$$z_m (\tau \to \infty) = z_m^* = \ln \left[1 + \frac{1}{(\theta_m - 1)\, a_1} \right]. \tag{14}$$

Expression (14) determines the limiting position of the isotherm θ_m. At $z > z_m^*$, the temperature of the medium does not exceed θ_m for any heating time. Because of this, the quantity z_m^* can be called the localization boundary of heating of the medium. Indeed, it follows from (11) that

$$\theta_a (\tau \to \infty) = 1 + \frac{1}{a_1} \frac{1}{\exp z - 1}, \quad \text{then} \quad \theta_a (\tau \to \infty, z = z_m^*) = \theta_m .$$

Results of a calculation of the temperature field by formulas (11)-(13) for $\theta_m = 1.4$, $\theta_l = 1.6$, $a_1 = 7.5$, $a_2 = 17.5$, $a_3 = 4$, and $a_4 = 0.5$ are presented in Fig. 3, from which it is seen that heating of the medium in the regime with localization occurs, the temperature remains constant in the region $z > z_m^*$, and it increases with time in the region $0 < z \le z_m^*$. For $z \to z_m^*$, a large (but finite) temperature gradient is realized. The heating in the regime with peaking, occurring as a result of the dependence $d\alpha/dT > 0$, is not able to manifest itself completely, since at $\theta = \theta_m$ the properties of the medium change and $d\alpha/dT < 0$. The temperature wave is localized in the region $z < z_m^*$, since at $\theta = \theta_l$ the properties of the medium change once again and in subsequent heating $\alpha = \alpha_l = \text{const}$. It follows from (14) that the coordinate of the boundary of the region of localization of heating is independent of the temperature and is determined by the parameters α_m and α_0. For $\alpha_0 \to \alpha_m$, i.e., at $a_1 = 0$, localization of heating is absent and $z_m^* \to \infty$.

5. We us consider the results of a numerical calculation of problem (5)-(6) for functions $f(z, \tau)$ determined in accordance with (7) and (10). In the numerical calculations we used an implicit difference scheme with a variable step along the coordinate with the same order of approximation of the equation and a boundary condition of the second kind at $x = 0$. Neglect of heat conduction in problems of heating with the use of volumetric heat sources is reliable for times [6] $t << h^2/a$. This condition can be represented in the form $\tau a_0 << 1$. For the parameters adopted above (with allowance for $\lambda = 1$ W/(m·K)) we have $a_0 \approx 0.0012$ and $\tau << 833$. The characteristic times of heating are of the order of $\tau \le 10$ ($t \le 6 \cdot 10^5$ sec). Then it can be expected that neglect of heat conduction is fully justified and the solutions obtained above in the approximation $a_0 = 0$ adequately describe actual processes. A comparison of the numerical calculations with expressions (8)-(9) confirmed this assumption: for times up to $\tau = 5$, the results of the numerical calculations and the analytical expressions are practically the same.

A different pattern is observed in the case of a nonmonotonic dependence of the absorption coefficient on the temperature (10). Results of numerical calculations performed for $a_1 = 45$, $a_2 = 47$, $a_3 = 10$, and $a_4 = 0.5$ are presented in Fig. 4. The fundamental difference between the results presented in Figs. 3 and 4 is that in a heat-conducting medium, localization of heating is absent. The heat conduction influences the quantitative characteristics only slightly (the temperature at a given point is practically the same with and without allowance for the heat conduction) but it changes essentially the qualitative characteristics of the process. As a result of transfer of heat by the mechanism of heat conduction, the region of the nonlinear dependence $\alpha(T)$ moves into the depth of the medium, with the result that a mechanism of volumetric heating that greatly surpasses the mechanism of heat conduction in intensity appears. Ultimately, a quasistationary temperature wave whose characteristics (amplitude and velocity) are practically independent of the heat conduction is formed. Thus, a small effect of heat conduction is favorable for realizing a large effect of volumetric heating.

The velocity of the temperature wave decreases with time, which is due to the infiniteness of the heat-source function ($\alpha_1 \neq 0$ and therefore $a_4 \neq 0$ in (10)). For $\alpha_1 \neq 0$, behind the temperature-wave front there is a heating region (where $\theta > \theta_1$) in which a portion of the radiation energy is absorbed and the motion of the temperature-wave front is slowed down. From the integral of the heat balance, an expression determining the deceleration of the temperature-wave front that is due to the infiniteness of the heat-source function follows:

$$\frac{dz_{1f}}{d\tau} - \frac{dz_1}{d\tau} = \frac{1}{\theta_1 - 1} \int_0^{z_1(\tau)} \frac{\partial \theta}{\partial \tau} dz, \quad z_{1f} = z_1 (\alpha_1 = 0) .$$

Substituting here the expression for θ from Eq. (5) with $f = a_4$, after some manipulation we obtain

$$\frac{dz_{1f}}{d\tau} = \frac{dz_1}{d\tau} + \frac{a_0}{\theta_1 - 1} \frac{\partial \theta (z_1, \tau)}{\partial z} + \frac{2}{\theta_1 - 1} [1 - \exp(-a_4 z_1)] .$$

Upon integration of this equation in the approximation $a_0 = 0$ with allowance for the fact that

$$\frac{2}{\theta_1 - 1} = V_s = \frac{dz_{1f}}{d\tau} ,$$

we have

$$z_1 = \frac{1}{a_4} \ln \left[1 + \frac{2a_4}{\theta_1 - 1} (\tau - \tau_s) \right] . \tag{15}$$

This expression determines the law of motion of the temperature-wave front, and τ_s is the time of wave formation determined from the condition $\theta(0, \tau_s) = \theta_1$. The values of z_1 calculated by the indicated formula practically coincide with the results of numerical calculations performed with allowance for the heat conduction, since the influence of the latter, resulting in some increase in the wave-front velocity, is negligibly small for actual times of the process. The velocity of the temperature wave determined from (15) is as follows in dimensional form:

$$V_s = \frac{q_0}{\rho c (T_1 - T_0) + 2\alpha_1 q_0 (t - t_s)}.$$

It is seen that the decrease in the velocity with time is due to the fact that $\alpha_1 \neq 0$. For $\alpha_1 = 0$, this expression yields the value of the velocity obtained in Sec. 3.

The numerical experiments have shown that the temperature wave possesses structural stability. Temperature disturbances of arbitrary amplitude arising at the initial or any subsequent moment of time have no influence on the structure of the temperature wave. At the initial moments the peak of a disturbance increases fairly rapidly (especially in the region of peaking $d\alpha/dT > 0$) and, as a result of heat conduction, some a spread of the disturbance in space occurs. When the temperature wave approaches, the fundamental amplitude of the disturbance peak is cut by the wave, which leaves behind a relatively small disturbance of the temperature profile, which disappears with time.

The investigated features of the temperature field observed in dissipation of energy of electromagnetic radiation energy into heat determine the possibility of control and optimization of the process of heating various media.

NOTATION

Q, density of the heat sources; α, q_0, and ω, absorption coefficient, intensity, and frequency of the electromagnetic radiation; c_0, velocity of light in vacuum; ε and $\tan\delta$, dielectric permittivity and tangent of the dielectric-loss angle of the medium; x, coordinate; t, time; T, temperature; h, depth of penetration of electromagnetic radiation into the medium; α_0, α_m, and a_1, characteristic absorption coefficients; T_0, T_m, and T_1, characteristic temperatures; λ, coefficient of thermal conductivity; ρc, heat capacity per unit volume of the medium; γ, γ_1, and γ_2, temperature coefficients of absorption; θ_s, z_s, and V_s, dimensionless amplitude (temperature), coordinate, and velocity of the temperature wave; θ, z, and τ, dimensionless temperature, coordinate, and time; $t_0 = \rho c T_0/\alpha_0 q_0$, characteristic time of heating; τ_m and τ_1, characteristic times; $z_m(\tau)$ and $z_1(\tau)$, dimensionless coordinates of the isotherms $\theta = \theta_m$ and $\theta = \theta_1$; z_m^*, localization boundary of heating; a, coefficient of thermal diffusivity of the medium; z_{1f} and z_1, coordinates of the temperature-wave front for $\alpha_1 = 0$ and $\alpha \neq 0$; τ_s, time of temperature-wave formation.

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